CHAPTER 9 INFINITE SERIES

9.1 SEQUENCES

OBJECTIVE A: Given a defining rule for the sequence $\{a_n\}$, write the first few items of the sequence.

OBJECTIVE B: Given a sequence $\{a_n\}$, determine if it converges or diverges. If it converges, use the theorems presented in this article of the text, or l'Hôpital's rule, or Table 9.1 to find the limit.

9.2 INFINITE SERIES

OBJECTIVE A: For a given geometric series $\sum_{n=1}^{\infty} ar^{n-1}$, determine if the series converges or diverges. If it does converge, then compute the sum of the series. The indexing of the series may be changed for a given problem.

OBJECTIVE B: Use the n^{th}-term test for divergence to test a given series $\sum_{n=1}^{\infty} a_n$ for divergence.

9.3 SERIES WITH NONNEGATIVE TERMS - LIMIT COMPARISON TEST

OBJECTIVE A: Know the Nondecreasing Sequence Theorem and how it applies to an infinite series of nonnegative terms.

9.4 SERIES WITH NONNEGATIVE TERMS—RATIO TEST

OBJECTIVE A: Given a series with nonnegative terms, investigate its convergence or divergence using the ratio test.

9.5 ALTERNATING SERIES AND ABSOLUTE CONVERGENCE

OBJECTIVE A: Use the Alternating Series Theorem (Leibniz's Theorem) to investigate the convergence of an alternating series.

OBJECTIVE B: Use the Alternating Series Estimation Theorem to estimate the magnitude of the error if the first k terms, for some specified number k, are used to approximate a given alternating series.

OBJECTIVE C: Given an infinite series, use the tests in Table 9.2 of the text to determine if the series is absolutely convergent, conditionally convergent, or divergent.

9.6 POWER SERIES

OBJECTIVE A: Given a power series $\sum_{n=0}^{\infty} c_n (x - a)^n$, find its interval of convergence. If the interval is finite, determine whether the series converges at each endpoint.

OBJECTIVE B: Given a power series $f(x) = \sum a_n x^n$, find the power series for $f'(x)$.

OBJECTIVE C: If f is a function having a known power series $f(x) = \sum a_n x^n$, use the series and a calculator to estimate the integral $\int_0^b f(x)\,dx$, assuming that b lies within the interval of convergence.

9.7 TAYLOR AND MACLAURIN SERIES

OBJECTIVE A: Find the Taylor Series at $x = a$, or the Maclaurin series, for a given function $y = f(x)$. Assume that $x = a$ is specified and that f has finite derivatives of all orders at $x = a$.

OBJECTIVE B: Know the statement of Taylor's Theorem and a formula for the remainder of order n.

OBJECTIVE C: Use the Remainder Estimation Theorem to estimate the truncation error when a Taylor polynomial is used to approximate a function.

9.8 CALCULATIONS WITH TAYLOR SERIES

OBJECTIVE A: Write the binomial series for functions $(1 + x)^m$ and know where it converges.

OBJECTIVE B: Use Taylor series to evaluate non-elementary integrals.

OBJECTIVE C: Use Taylor series to approximate the value of an integral.

OBJECTIVE D: Use Taylor series to solve initial value problems.

CHAPTER 10 CONIC SECTIONS, PARAMETERIZED CURVES AND POLAR COORDINATES

10.1 CONIC SECTIONS AND QUADRATIC EQUATION

OBJECTIVE A: Given an equation of an ellipse $Ax^2 + Cy^2 = F$, where A, C and F are positive numbers, put the equation in standard form and find the ellipse's eccentricity. Sketch the ellipse and include the foci in your sketch.

OBJECTIVE B: Given an equation of a hyperbola $Ax^2 - Cy^2 = F$ or $Cy^2 - Ax^2 = F$, where A, C and F are positive numbers, put the equation in standard form and find the hyperbola's eccentricity and asymptotes. Sketch the hyperbola, including the asymptotes and foci in your sketch.

10.3 PARAMETERIZATIONS OF CURVES

OBJECTIVE A: Given parametric equations $x = f(t)$ and $y = g(t)$ for the motion of a particle in the xy-plane, eliminate the parameter t to find a Cartesian equation for the particle's path. Graph the Cartesian equation.

OBJECTIVE B: Find parametric equations for a curve described geometrically, or by an equation, in terms of some specified or arbitrary parameter.

10.4 CALCULUS WITH PARAMETERIZED CURVES

OBJECTIVE A: Given parametric equations $x = f(t)$ and $y = g(t)$, find dy/dx in terms of dy/dt and dx/dt. Find d^2y/dx^2 in terms of t.

OBJECTIVE B: Find the length of a smooth curve specified parametrically by continuously differentiable equations $x = f(t)$ and $y = g(t)$ over a given interval $a \leq t \leq b$.

10.5 POLAR COORDINATES

OBJECTIVE A: Given a point P in polar coordinates $(r; \theta)$, give the Cartesian coordinates $(x; y)$ of P.

OBJECTIVE B: Graph the points $P(r; \theta)$ whose polar coordinates satisfy a given equation, inequality or inequalities.

OBJECTIVE C: Given an equation in polar coordinates, replace it by an equivalent equation in Cartesian coordinates and identify the graph.

10.6 POLAR GRAPHS

OBJECTIVE A: Given an equation $F(r; \theta) = 0$ in polar coordinates, analyze and sketch its graph.

OBJECTIVE B: If $r = f(\theta)$ is differentiable, find the slope dy/dx at the point $(r; \theta)$ on the graph of f.

CHAPTER 11 VECTORS AND ANALYTIC GEOMETRY IN SPACE

11.1 VECTORS IN THE PLANE

OBJECTIVE A: Given the points P_1 and P_2 in the plane, express the vector $\overrightarrow{P_1P_2}$ in the form $a\mathbf{i} + b\mathbf{j}$.

OBJECTIVE B: Express the sum and difference of two given vectors, and multiples of given vectors by scalars, in the form $a\mathbf{i} + b\mathbf{j}$.

OBJECTIVE C: Given a vector $\mathbf{v} = a\mathbf{i} = b\mathbf{j}$, calculate its length or magnitude, direction, and the angle it makes with the positive x-axis.

OBJECTIVE D: Find unit vectors tangent and normal to a given curve $y = f(x)$ at a specified point $P(a; b)$.

11.2 CARTESIAN (RECTANGULAR) COORDINATES AND VECTORS IN SPACE

OBJECTIVE A: Given two points P_1 and P_2 in space, express the vector $\overrightarrow{P_1P_2}$ from P_1 to P_2 in the form $a\vec{i} + b\vec{j} + c\vec{k}$.

OBJECTIVE B: Find the length of any space vector.

OBJECTIVE C: Given a Cartesian equation of a sphere in space, find the coordinates of its center and the radius.

OBJECTIVE D: Given a nonzero space vector find its direction.

11.3 DOT PRODUCTS

OBJECTIVE A: Find the scalar product or dot product of two vectors in space and the cosine of the angle between them.

OBJECTIVE B: Given two space vectors $\vec{A}$ and $\vec{B}$ find the projection vector $\text{proj}_{\vec{A}}\,\vec{B}$ and the scalar component of $\vec{B}$ in the direction $\vec{A}$.

OBJECTIVE C: Use the vector methods to calculate the distance between a given point S and a given line $L : Ax + By = C$ in the $xy-$plane.

OBJECTIVE D: Know the properties of the scalar product.

OBJECTIVE E: Find an equation for the line in the $xy-$plane that passes through a given point $P(x_0, y_0)$ and is perpendicular to a specified vector $\vec{N} = A\vec{i} + B\vec{j}$.

11.4 CROSS PRODUCTS

OBJECTIVE A: Define the vector product or cross product of two vectors in space, and give at least five properties of the cross product.

OBJECTIVE B: Use the determinant formula to calculate the cross product of any two vectors in space whose $\vec{i}, \vec{j}, \vec{k}$ components are given.

OBJECTIVE C: Find a vector that is perpendicular to two given vectors in space.

OBJECTIVE D: Find the area of any triangle with specified vertices in space, and find the distance between the origin and the plane determined by that triangle.

11.5 LINES AND PLANES IN SPACE

OBJECTIVE A: Write parametric equations of a line in space given (a) two points on the line, or (b) a point on the line and a vector parallel to the line.

OBJECTIVE B: Find the distance between a given point S and line L in space.

OBJECTIVE C: Write an equation of a plane in space given (a) a point on the plane and a vector normal to the plane, or (b) three noncollinear points on the plane.

OBJECTIVE D: Find the distance between a given point S and plane in space.

OBJECTIVE E: Find the point in which a given line meets a given plane.

OBJECTIVE F: Find parametric equations for the line in which two given nonparallel planes intersect. Find also the (acute) angle between the planes.

11.6 SURFACES IN SPACE

OBJECTIVE A: Discuss and sketch cylinders whose equations are given.

OBJECTIVE B: Discuss and sketch a given surface whose equation $F(x, y, z) = 0$ is a quadratic in the variables x, y, and z.

11.7 CYLINDRICAL AND SPHERICAL COORDINATES

OBJECTIVE A: Describe the set of points in space whose Cartesian, cylindrical, or spherical coordinates satisfy given pairs of simultaneous equations.

OBJECTIVE B: Translate an equation from a given coordinate system (Cartesian, cylindrical, or spherical) into forms that are appropriate to the other two systems.

CHAPTER 12 VECTOR-VALUED FUNCTIONS

12.1 VECTOR-VALUED FUNCTIONS AND SPACE CURVES

OBJECTIVE A: Find the derivative of a given vector function and give the domain of the derived function.

OBJECTIVE B: Given the position vector of a particle at any time t, find the velocity and acceleration vectors. Evaluate these vectors and find the speed and direction of motion of the particle at any instant of time.

OBJECTIVE C: Using the appropriate rules, calculate the derivatives of vector expressions involving the dot or cross products.

OBJECTIVE D: Find the definite or indefinite integral of a vector-valued function having continuous components.

OBJECTIVE E: Find the position vector $\mathbf{r}(t)$ when the acceleration vector $\mathbf{a}(t)$ is known together with initial conditions $\mathbf{v}(0)$ and $\mathbf{r}(0)$. That is, solve a vector-valued initial value problem.

12.3 ARC LENGTH AND THE UNIT TANGENT VECTOR $\mathbf{T}$

OBJECTIVE A: Given the coordinates for a curve in space in terms of some parameter t, find the length of the curve for a specified interval $a \le t \le b$.

OBJECTIVE B: Give the position vector for the motion of a particle find the unit tangent vector $\mathbf{T}$ to the curve at any point of the curve.

CHAPTER 13 PARTIAL DERIVATIVES

13.1 FUNCTIONS OF SEVERAL INDEPENDENT VARIABLES

OBJECTIVE A: Given a function $w = f(x; y)$, ¯nd its domain and range.

OBJECTIVE B: Given a function $w = f(x; y)$,represent the function (a) by sketching a surface in space, and (b) by drawing an assortment of level curves in the plane.

OBJECTIVE C: Find an equation for a level surface of a given function $f(x; y; z)$ that passes through a speci¯ed point $(x_0; y_0; z_0)$.

13.2 LIMITS AND CONTINUITY

OBJECTIVE A: Given an elementary function of two variables x and y, ¯nd its limit as $(x; y)$ approaches the point $(x_0; y_0)$, if the limit exists.

OBJECTIVE B: Determine the points in the xy ¡ plane at which a given function $f(x; y)$ is continuous.

13.3 PARTIAL DERIVATIVES

OBJECTIVE A: Given an equation of a real-valued function of several variables, ¯nd the partial derivatives with respect to each variable.

OBJECTIVE B: Given a function of several independent variables, calculate all partial derivatives of the second order.

13.4 DIFFERENTIABILITY, LINEARIZATION, AND DIFFERENTIALS

OBJECTIVE A: Given a function $f(x; y)$, ¯nd the standard linear approximation to it near a speci¯ed point.

OBJECTIVE B: Given a surface $w = f(x; y)$, ¯nd the change df given by the linearization of f if we move $(x_0; y_0)$ to a nearby point $(x_0 + dx; y_0 + dy)$ for speci¯ed di®erentials dx and dy.

OBJECTIVE C: Use Equation (12) in the text to ¯nd an upper bound for the magnitude jEj of the error in the approximation $f(x; y)$ ¼ $L(x; y)$ over a speci¯ed rectangle R.

OBJECTIVE D: Use the approximation ¢ $f = f(x; y)$ ¡ $f(x_0; y_0)$ ¼ $f_x(x_0; y_0)\,dx + f_y(x_0; y_0)\,dy = df$ to discuss the sensitivity of $f(x; y)$ to small changes in x and y near a given point $(x_0; y_0)$.

13.5 THE CHAIN RULE

OBJECTIVE A: Let w be a di®erentiable function of the variables $x; y : : : ; v$ and let these in turn be di®erentiable functions of a second set of variables $p; q; : : : ; t$. Calculate the derivative of w with respect to any one of the variables in the second set by use of the chain rule for partial derivatives.

OBJECTIVE B: Assuming a given equation $F(x, y, z) = 0$ determine z as a differentiable function of x and y, calculate the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$ at points where $F_z \neq 0$.

13.6 DIRECTIONAL DERIVATIVES, GRADIENT VECTORS, AND TANGENT PLANES

OBJECTIVE A: Given a function f of two or three variables, find the gradient vector at a specified point.

OBJECTIVE B: Given a function f of two or three variables, find the directional derivative of f at a given point, and in the direction of a given vector $\vec{A}$.

OBJECTIVE C: Given a function f of two or three variables, determine the direction one should travel, starting from a given point P_0, to obtain the most rapid rate of increase or decrease (whichever is specified) of the function.

OBJECTIVE D: Find the plane which is tangent to the level surface $f(x, y, z) = \text{constant}$ at a specified point $P_0(x_0, y_0, z_0)$.

OBJECTIVE E: Given a surface $w = f(x, y)$, or $g(x, y, z) = 0$, find an equation of the tangent plane to the surface at a specified point P_0, if the tangent plane exists.

OBJECTIVE F: Given a surface $w = f(x, y)$, or $g(x, y, z) = 0$, find the normal line (if it exists) to the surface at a specified point P_0.

OBJECTIVE G: Given two surfaces $f(x, y, z) = \text{constant}$ and $g(x, y, z) = \text{constant}$, find parametric equations for the line tangent to the curve C of intersection of the surfaces at a specified point $P_0(x_0, y_0 z_0)$ on C.

13.7 MAXIMA, MINIMA, AND SADDLE POINTS

OBJECTIVE A: Given the surface $z = f(x, y)$ defined by a function f which has continuous partial derivatives over some regions R, examine the surface for local extrema. Use the second derivative test to classify the relative extrema as local maxima, local minima, or saddle points.

OBJECTIVE B: Find the absolute maxima and minima of a given function $f(x, y)$ over a specified domain.

CHAPTER 14 MULTIPLE INTEGRALS

14.1 DOUBLE INTEGRALS

OBJECTIVE A: Evaluate a given iterated double integral and sketch the region over which the integration extends.

OBJECTIVE B: Given a (double) iterated integral, write an equivalent double iterated integral with the order of integration reversed. Sketch the region over which the integration takes place and evaluate the new integral.

OBJECTIVE C: Find the volume of solid whose base is a specified region A in the $xy-$plane, and whose top is a given surface $z = f(x, y)$.

14.2 AREAS, MOMENTS, AND CENTERS OF MASS

OBJECTIVE A: For a speci¯ed region R in the xy ¡ plane (a) sketch the region R; (b) label each bounding curve of the region with its equation, and ¯nd the coordinates of the boundary points where the curves intersect; and (c) ¯nd the area of the region by evaluating an appropriate (double) iterated integral.

OBJECTIVE B: Given a plane region R in the xy ¡ plane, ¯nd its center of mass.

14.3 DOUBLE INTEGRALS IN POLAR FORM

OBJECTIVE A: Change a given Cartesian integral to an equivalent double integral in polar coordinates and then evaluate the polar integral.

OBJECTIVE B: In an applied problem involving double integration (e.g., ¯nding an area, volume) express the double integral in polar coordinates (when appropriate, to make the integrations easier), and then evaluate the integral thus obtained.

14.4 TRIPLE INTEGRALS IN RECTANGULAR COORDINATES

OBJECTIVE A: Evaluate a given iterated triple integral

OBJECTIVE B: By triple integration, ¯nd the volume of a speci¯ed region D in xyz ¡ space.

OBJECTIVE C: Find the average value of a given function F(x; y; z) over a speci¯ed region in xyz ¡ space.

14.7 SUBSTITUTION

OBJECTIVE: Change double and triple integrals from one set of variables to another by substitution (using the Jacobian of the transformation) and evaluate those integrals.